

# Simulation of Five-Step One-Bed Sorption-Enhanced Reaction Process

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*A five-step, one-bed, sorption-enhanced reaction process proposed by Carvill et al. in 1996 for hydrogen production by steam–methane reforming was analyzed. For the simulated results of the first step, data from Hufton et al. (1999) and Ding and Alpay (2000) for a fixed-bed column of an admixture of a catalyst and a sorbent that selectively removes CO<sub>2</sub> from the reaction zone were used. The sorbent is periodically regenerated by using the principles of pressure-swing adsorption. The process steps allow direct production of hydrogen with high purity and high methane conversion. A model considering multicomponent and overall mass balance, Ergun relation for pressure drop, energy balance for the bed-volume element, and nonlinear adsorption equilibrium isotherm coupled with three main reactions was derived to describe the sorption-enhanced reaction cyclic process. Two different isotherms were used under wet and dry conditions. The LDF model adopted describes the mass-transfer rate of CO<sub>2</sub> in the adsorbent. Numerical solution of model equations for the cyclic process was obtained by the orthogonal collocation method. The operating conditions allow the combination of a sufficiently high purity of hydrogen (average purity over 80%) with traces of CO and CO<sub>2</sub>, high methane conversion, fast adsorbent regeneration, and cyclic steady-state operation. The model predictions agree reasonably with the literature data. The package is suitable for the design and analysis of sorption-enhanced reaction process.*

## Introduction

Hydrogen is used extensively by many industries for a variety of applications; it is produced industrially by several methods, such as steam reforming, electrolysis of water, ammonia dissociation, and partial oxidation. In addition, hydrogen is obtained as a byproduct of some refining and chemical production processes. Manufacture via steam reforming of natural gas is a low-cost option for hydrogen production (Ridler and Twigg, 1989).

Many reactions are limited by equilibrium, and the production of hydrogen by steam–methane reforming is a good example. The combination of reaction and separation in a single unit operation has advantages for achieving enhanced conversions and yields in catalyzed reversible reactions; this concept is based on the well-known Le Chatelier's principle that (1) the conversion of reactants to products, and (2) the rate of forward reaction in an equilibrium-limited reaction can be increased by selectively removing some of the reaction products from the reaction zone (Gluud et al., 1931).

Configurations under investigation include selective adsorption with reaction (Roginskii et al., 1962; Magee, 1963; Chu and Tsang, 1971; Takeuchi and Uruguchi, 1977; Cho et

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al., 1980, 1982; Petroulas et al., 1985; Fish et al., 1986; Tonkovich and Carr, 1994a,b; Kruglov, 1994; Lu and Rodrigues, 1994; Carvill et al., 1996; Hufton et al., 1999, 2000; Ding and Alpay, 2000b), selective permeation with reaction (Sun and Khang, 1988; Hsieh, 1989; Champagnie et al., 1990; Wu et al., 1990; Uemiya et al., 1991; Tsotsis et al., 1992; Itoh et al., 1993; Adris et al., 1994; Armor, 1995; Balachandran et al., 1997; Omorjan et al., 1999; Ostrowski and Mleczko, 1999), and imposed thermal effects (Rostrup-Nielsen, 1993; Jachuck and Ramshaw, 1994; Matros and Bunimovich, 1995; Salinger and Eigenberger, 1996; Yongsunthorn and Alpay, 1998, 1999). Of particular interest is a pressure swing adsorptive (PSA) reactor.

If an adsorbent selectively trapping the product is combined with the catalyst, conversion can almost run to completion. The PSA reactor, in which a high-pressure feed of synthesis gas to a mixed bed of adsorbent and catalyst (reaction and adsorption of  $\text{CO}_2$ ) is combined with rapid pressure release to low pressure ( $\text{CO}_2$  desorption), is one reasonable configuration for such a production process (Carvill et al., 1996; Hufton et al., 1999, 2000; Ding and Alpay, 2000b).

From the point of view of application, two key problems should be solved for hydrogen production by sorption-enhanced steam-methane reforming process: (1) to find suitable adsorbent with high adsorption capacity, high mass-transfer rates for both adsorption and desorption of  $\text{CO}_2$  for its removal from the reaction zone, and regeneration; (2) to seek out a simplistic and energy-saving cyclic process for adsorbent regeneration. Because of the high complexity of the process, the design and optimization of the PSA reactor based only on experiments is not possible.

There are a large number of adsorbents for  $\text{CO}_2$  removal, which include various metal oxides (e.g.,  $\text{CaO}$ ,  $\text{MgO}$ ), alumina and metal-promoted alumina, activated carbon, and zeolites (e.g., 4A, 5A, CrA, CrX, CrY, RhA, 13X, and Na- and H-mordenites) (Ma and Mancel, 1972; Ma and Roux, 1973; Hayhurst, 1980; Wilson and Danner, 1983; Valenzuela and Myers, 1989; Han and Harrison, 1994; Anand et al., 1995; Hufton et al., 1999; Ding and Alpay, 2000a).

The stability of the adsorbent under cyclic reactor conditions, that is, under constant temporal and spatial variations of gas composition and temperature, will govern the commercial feasibility. Most recently  $\text{CO}_2$  adsorption capacity on hydrotalcite has been presented (Hufton et al., 1999; Ding and Alpay, 2000a). Hydrotalcite is an anionic clay consisting of positively charged layers of metal oxide (or metal hydroxide) with interlayers of anions, such as carbonate. Exchange of the metal cations, as well as intercalation of the anionic layer can lead to a wide range of catalytic and adsorptive properties, with particular stability under wet gas and high-temperature conditions (Hufton et al., 1999; Ding and Alpay, 2000a). Thus, such materials are potentially suitable for the separation-enhanced steam-methane reforming process, and these types of adsorbents are commercially available now (see Zou et al., 2001).

A two-, three-, four-, or five-step simplistic process is considered by most of the published works theoretically and experimentally. Kadlec and coworkers experimentally (Vaporciyan and Kadlec, 1989) and theoretically (Vaporciyan and Kadlec, 1987; Lee and Kadlec, 1989) evaluated a three-step process consisting of (1) introduction of the reactant-feedgas mixture into a packed bed of catalyst and

adsorbent at an elevated pressure for a short period of time, followed by (2) a short delay period when the introduction of feedgas was stopped, and finally (3) countercurrent depressurization of the packed bed to the lowest pressure level of operation.

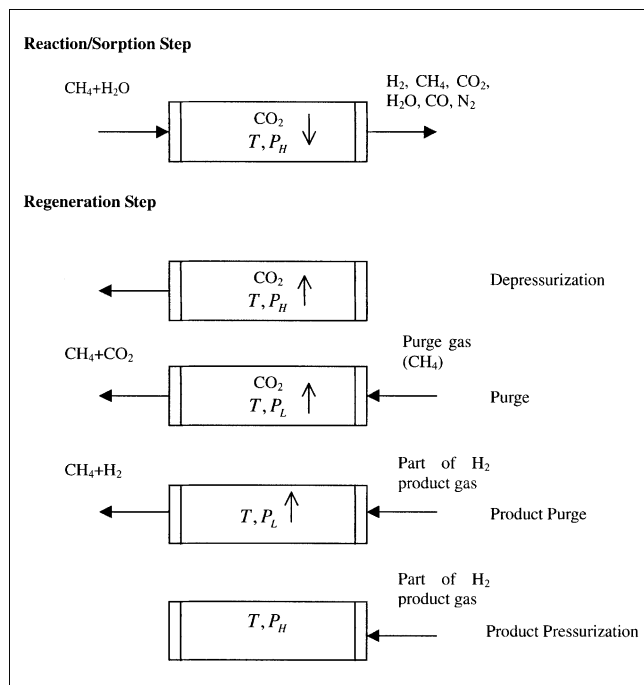
Alpay and coworkers (Alpay et al., 1993, 1994; Chatsiriwech et al., 1994; Kirkby and Morgan, 1994; Ding and Alpay, 2000b) evaluated model solutions of the PSA reactor system using a process similar to that just mentioned except where step (2) was eliminated (Alpay et al., 1993). They also theoretically evaluated a four-step process (Alpay et al., 1994; Chatsiriwech et al., 1994; Kirkby and Morgan, 1994) consisting of (1) pressurizing the packed bed of catalyst and adsorbent with the reactant-feed mixture to a higher pressure level, (2) feeding the reactant mixture at the elevated pressure and withdrawing a product gas, (3) countercurrently depressurizing the reactor to the lowest pressure level of operation, and (4) countercurrently purging the reactor with the effluent from step 2. A variation of this cycle included partial countercurrent pressurization of the reactor with the effluent from step 2 before step 1 started (Kirkby and Morgan, 1994).

Sircar and coworkers (Carvill et al., 1996) considered a general equilibrium-controlled reaction involving one or more reactants producing a desired reaction product and a byproduct. They proposed a five-step one-bed process, and used the reverse water-gas shift reaction to produce high-purity CO as an example. They applied this concept to hydrogen production by the steam-methane reforming process (Hufton et al., 1999), and later improved the process cycle suggesting a four-step one-bed process for hydrogen production (Hufton et al., 2000).

In this article, a five-step (high-pressure reaction/adsorption, depressurization, low-pressure purge with methane, low-pressure purge with part of hydrogen product, and pressurization with part of hydrogen product) one-bed sorption-enhanced reaction process (proposed by Carvill et al., 1996) was theoretically analyzed for hydrogen production by steam-methane reforming. The simulated data for the first step use data from the articles of Hufton et al. (1999) and Ding and Alpay (2000a,b). A model that takes into account multicomponent (six species) mass balances, overall mass balance, Ergun relation (Ergun, 1952) for pressure drop, energy balance for bed-volume element including the heat-transfer to the column wall, and nonlinear adsorption equilibrium isotherm coupled with the three main reactions was derived to describe the sorption-enhanced reaction cyclic process. Two different isotherms were used under wet and dry conditions; the linear driving-force (LDF) model was adopted to describe the mass-transfer rate of  $\text{CO}_2$  to the adsorbent. A numerical solution of the model equations for the cyclic process was obtained by using the method of orthogonal collocation (Malek and Farooq, 1997). The purpose of this work is to find suitable operating conditions that allow the production of a sufficiently high hydrogen purity (average purity over 80%) with traces of CO and  $\text{CO}_2$ , high methane conversion, fast adsorbent regeneration, and cyclic steady-state operation.

## Process Description

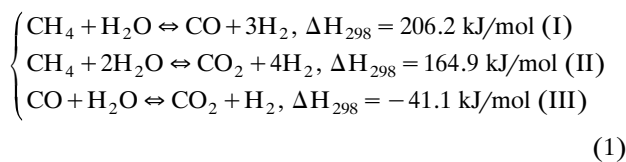
The operation scheme of the five-step one-bed sorption-enhanced reaction process is shown in Figure 1. The three



**Figure 1. Flow direction for five-step one-bed sorption-enhanced reaction process.**

↓ means adsorption; ↑ means desorption.

main chemical reactions in the cycle are described by Eq. 1 (Xu and Froment, 1989)



Only two of the three reactions are linearly independent. The reaction kinetic model of Xu and Froment (1989) can be summarized as

$$R_I = \frac{1}{(DEN)^2} \frac{k_1}{P_{\text{H}_2}^{2.5}} \left( P_{\text{CH}_4} P_{\text{H}_2\text{O}} - \frac{P_{\text{H}_2}^3 P_{\text{CO}}}{K_I} \right) \quad (1a)$$

$$R_{II} = \frac{1}{(DEN)^2} \frac{k_2}{P_{\text{H}_2}^{3.5}} \left( P_{\text{CH}_4} P_{\text{H}_2\text{O}}^2 - \frac{P_{\text{H}_2}^4 P_{\text{CO}_2}}{K_{II}} \right) \quad (1b)$$

$$R_{III} = \frac{1}{(DEN)^2} \frac{k_3}{P_{\text{H}_2}} \left( P_{\text{CO}} P_{\text{H}_2\text{O}} - \frac{P_{\text{H}_2} P_{\text{CO}_2}}{K_{III}} \right), \quad (1c)$$

where  $DEN = 1 + K_{\text{CO}} P_{\text{CO}} + K_{\text{H}_2} P_{\text{H}_2} + K_{\text{CH}_4} P_{\text{CH}_4} + K_{\text{H}_2\text{O}} P_{\text{H}_2\text{O}}/P_{\text{H}_2}$ , in which  $P_i = y_i P$  ( $i = \text{H}_2\text{O}, \text{CH}_4, \text{H}_2, \text{CO}_2$ , and  $\text{CO}$ ,  $P$  is the local total pressure, and  $y_i$  is the gas-phase mole fraction of component  $i$ ). For the expressions of  $k_1, k_2, k_3, K_{\text{CO}}, K_{\text{H}_2}, K_{\text{CH}_4}$  and  $K_{\text{H}_2\text{O}}$  and  $K_I, K_{II}$  and  $K_{III}$ , see Table 1.

The equilibrium data  $K_I, K_{II}$ , and  $K_{III}$  are the key parameters for our simulation. There are two sources (Xu and Froment, 1989; Twigg, 1989) from which to obtain their expressions, as listed in Table 1; for our work, we take the results of Twigg (1989).

The formation or consumption rate of component  $i$ ,  $r_i$ , was then calculated by using Eqs. 1a–1c as follows:

$$r_i = \sum_{j=I}^{III} v_{ij} R_j \quad (i = \text{CH}_4, \text{H}_2\text{O}, \text{H}_2, \text{CO}_2, \text{CO}), \quad (2a)$$

where  $v_{ij}$  is the stoichiometric coefficient of component  $i$  in reaction  $j$ . If  $i$  refers to a reactant,  $v_{ij}$  is negative, and for a

**Table 1. Parameters Used in Eq. 1**

|  |  |
|--|--|
| $K_I = \frac{1}{\exp(0.2513Z^4 - 0.3665Z^3 - 0.58101Z^2 + 27.1337Z - 3.2770)} \text{ atm}^2, ^* K_{II} = K_I K_{III}$  |  |
| $K_{III} = \exp(-0.29353Z^3 + 0.63508Z^2 + 4.1778Z + 0.31688), ^{**} \text{ where } Z = \frac{1,000}{T} - 1$   |  |
| $k_1 = 1.842 \times 10^{-4} \exp \left[ -\frac{240,100}{R} \left( \frac{1}{T} - \frac{1}{648} \right) \right] \text{ kmol} \cdot \text{bar}^{0.5} / \text{kg}_{\text{cat}} \cdot \text{h}$ |  |
| $k_2 = 2.193 \times 10^{-5} \exp \left[ -\frac{243,900}{R} \left( \frac{1}{T} - \frac{1}{648} \right) \right] \text{ kmol} \cdot \text{bar}^{0.5} / \text{kg}_{\text{cat}} \cdot \text{h}$ |  |
| $k_3 = 7.558 \exp \left[ -\frac{67,130}{R} \left( \frac{1}{T} - \frac{1}{648} \right) \right] \text{ kmol} / \text{kg}_{\text{cat}} \cdot \text{h} \cdot \text{bar}$                       |  |
| $K_{\text{CH}_4} = 0.179 \exp \left[ \frac{38,280}{R} \left( \frac{1}{T} - \frac{1}{823} \right) \right] \text{ bar}^{-1},$  | $K_{\text{H}_2\text{O}} = 0.4152 \exp \left[ -\frac{88,680}{R} \left( \frac{1}{T} - \frac{1}{823} \right) \right]$         |
| $K_{\text{CO}} = 40.91 \exp \left[ \frac{70,650}{R} \left( \frac{1}{T} - \frac{1}{648} \right) \right] \text{ bar}^{-1},$  | $K_{\text{H}_2} = 0.0296 \exp \left[ \frac{82,900}{R} \left( \frac{1}{T} - \frac{1}{648} \right) \right] \text{ bar}^{-1}$ |

Source: Xu and Froment, 1989; Twigg, 1989.

\*, \*\* are taken from Twigg (1989). The data of Xu and Froment (1989) are as follows:  $K_I = 4.707 \times 10^{12} \exp \left( -\frac{224,000}{RT} \right) \text{ bar}^2$ ,  $K_{III} = 1.142 \times 10^{-2} \exp \left( \frac{37,300}{RT} \right)$  (for  $T = 948 \text{ K}$ ).

product  $v_{ij}$  is positive. Thus, we have

$$r_{\text{CH}_4} = -R_I - R_{II} \quad (2b)$$

$$r_{\text{H}_2\text{O}} = -R_I - 2R_{II} - R_{III} \quad (2c)$$

$$r_{\text{H}_2} = 3R_I + 4R_{II} + R_{III} \quad (2d)$$

$$r_{\text{CO}} = R_I - R_{III} \quad (2e)$$

$$r_{\text{CO}_2} = R_{II} + R_{III} \quad (2f)$$

Reforming reactions (I) and (II) are strongly endothermic, so the forward reaction is favored by high temperatures, while the water-gas shift reaction (III) is moderately exothermic and is therefore favored by low temperatures. The reforming reactions will also be favored at low pressures, whereas the water-gas shift reaction is largely unaffected by changes in pressure. In the presence of a selective  $\text{CO}_2$  adsorbent, the conversion of  $\text{CH}_4$  to  $\text{CO}_2$  is favored. For a reaction that is not kinetically limited, the use of an adsorbent will thus enable a lower operating temperature for a desired conversion. However, the separation effect is lost on equilibrium of the adsorbent. This is why the periodic regeneration of the adsorbent is needed.

The five steps of a one-bed sorption-enhanced reaction process are described below (Carvill et al., 1996) for our case.

**Step 1: High-Pressure Reaction/Adsorption.** Feed a stoichiometric mixture of  $\text{H}_2\text{O}$  and  $\text{CH}_4$  at high pressure  $P_H$  through the regenerated reactor that has been previously saturated

with part of the  $\text{H}_2$  product gas at  $P_H$ . An effluent stream containing essentially  $\text{H}_2$  is produced from the reactor at pressure  $P_H$ . The step is continued until near breakthrough point of  $\text{CO}_2$ .

**Step 2: Countercurrent Blowdown Step.** Depressurize the reactor to a lower pressure level of  $P_L$  countercurrent to that of the reactant-feed-gas flow. A gas stream containing all the components of the system exits the reactor. This stream consists of interparticle void gas in the column as well as some adsorbed gases present in the reactor at the end of Step 1. The reaction and desorption of  $\text{CO}_2$  should take place in the countercurrent blowdown step.

**Step 3: Low-Pressure Purge with Methane.** Introduce methane to the reactor at  $P_L$  in the direction countercurrent to that of the reactant feedgas flow. This step desorbs most of the remaining adsorbed  $\text{CO}_2$ , and the reactor effluent gas contains a mixture of  $\text{CH}_4$  and  $\text{CO}_2$ . The reactor is essentially saturated with  $\text{CH}_4$  at  $P_L$  at the end of this step.

**Step 4: Low-Pressure Purge with Part of the Hydrogen Product Gas.** Purge the reactor countercurrently at  $P_L$  with  $\text{H}_2$  in order to remove  $\text{CH}_4$  from the reactor void space and further desorb  $\text{CO}_2$  from the adsorbent. After two purge steps, the adsorbent is regenerated, and the remaining carbon dioxide concentration in the fixed-bed reactor is low enough for the next cycle.

**Step 5: Countercurrent Repressurization with Part of Hydrogen Product.** Pressurize the reactor from  $P_L$  to  $P_H$  by countercurrently introducing part of the  $\text{H}_2$  product gas from Step 1. During the countercurrent repressurization step, a part of the

**Table 2. Parameters for Governing Equations**

|   |   |
|---|---|
| Semi-empirical relationships for $K_D$ and $K_V$ (Ergun, 1952):   |   |
| $K_D = \frac{150\mu(1-\epsilon_b)^2}{d_p^2\epsilon_b^3} (\text{N}\cdot\text{s}/\text{m}^4)$ ,   | $K_V = \frac{1.75(1-\epsilon_b)}{d_p\epsilon_b^3} \frac{PM}{RT} (\text{N}\cdot\text{s}^2/\text{m}^5)$ , in which $M \approx 0.018 \text{ kg/mol}$ |
| Axial dispersion coefficient $D_L$ (Edwards and Richardson, 1968)   |   |
| $D_L = 0.73D_m + \frac{0.5ud_p}{1+9.49D_m/(ud_p)} (\text{m}^2/\text{s})$ , in which $D_m = 1.6 \times 10^{-5} \text{ m}^2/\text{s}$ for $P_H$ and $T_f$ , and   |   |
| $D_m = 5.6 \times 10^{-5} \text{ m}^2/\text{s}$ for $P_L$ and $T_f$ (Reid et al., 1988)   |   |
| Langmuir isotherm (Ding and Alpay, 2000a)   |   |
| $q_{\text{CO}_2}^* = \frac{m_{\text{CO}_2} b_{\text{CO}_2} P_{\text{CO}_2}}{1 + b_{\text{CO}_2} P_{\text{CO}_2}}$ , where $m_{\text{CO}_2} = 0.65 \text{ mol/kg}$ , $b_{\text{CO}_2} = 2.36 \times 10^{-4} \exp \left[ \frac{17,000}{R} \left( \frac{1}{T} - \frac{1}{673} \right) \right] \text{ Pa}^{-1}$ |   |
| (wet condition); $m_{\text{CO}_2} = 0.63 \text{ mol/kg}$ , $b_{\text{CO}_2} = 1.69 \times 10^{-4} \exp \left[ \frac{10,000}{R} \left( \frac{1}{T} - \frac{1}{673} \right) \right] \text{ Pa}^{-1}$ (dry condition)  |   |
| Mass-transfer coefficient (Ding and Alpay, 2000a)   |   |
| $k_{\text{CO}_2} = \frac{15}{r_p^2} \frac{\epsilon_p D_p}{\epsilon_p + \rho_p RT (\partial q_{\text{CO}_2}^* / \partial P_{\text{CO}_2})}$ , in which $\rho_p = 1,300 \text{ kg/m}^3$ , $D_p = 3.3 \times 10^{-7} \text{ m}^2/\text{s}$   |   |
| Bed effective conductivity $k_z$ (Yagi et al., 1960; Malek and Farooq, 1997)  |   |
| $\frac{k_z}{k_g} = \frac{k_z^0}{k_g} + 0.75(Pr)(Re_p)$ , where $\frac{k_z^0}{k_g} = \epsilon_b + \frac{\epsilon_b}{0.139\epsilon_b - 0.0339 + 2/3(k_g/k_p)}$ , $Pr = \frac{C_{pg}\mu}{k_g}$ , and   |   |
| $Re_p = \frac{\rho_g u \epsilon_b d_p}{\mu}$ , in which $k_p = 1 \times 10^{-2} \text{ J/cm}\cdot\text{s}\cdot\text{K}$ and $k_g = 2.5 \times 10^{-4} \text{ J/cm}\cdot\text{s}\cdot\text{K}$   |   |
| (Eucken formula, in Bird et al., 1960)  |   |
| Wall-bed heat-transfer coefficient, $U$ (Li and Finlayson, 1977)  |   |
| $\frac{2UR_0}{k_g} = 2.03Re_p^{0.8} \exp \left( -\frac{3d_p}{R_0} \right) \left( Re_p = 20 - 7,600, \frac{d_p}{2R_0} = 0.05 - 0.3 \right)$ , and  |   |
| $U = 6.15 \frac{k_z^0}{2R_0}$ as $Re_p \rightarrow 0$ (De Wasch and Froment, 1972)  |   |

remaining CO<sub>2</sub> in the gas phase is adsorbed by the adsorbent.

During Steps 1 to 5, the feedgas temperature  $T_f$  and the wall temperature  $T_w$  are kept constant.

## Theoretical Model

The theoretical model adopted for the sorption-enhanced reaction cyclic process is a nonisothermal, nonadiabatic, and nonisobaric operation, developed to describe both the steam-methane reforming (SMR) and the sorption-enhanced SMR (SE-SMR) processes. For the SE-SMR process, the reactions were assumed to take place on the surface of the catalyst. The model assumptions adopted are summarized below (Malek and Farooq, 1997; Silva and Rodrigues, 1998; Ding and Alpay, 2000b; Da Silva and Rodrigues, 2001):

1. The flow is represented by an axial-dispersed plug-flow model. Mass dispersion in the axial direction is considered, with negligible radial gradients. The axial dispersion coefficient was estimated from the correlation of Edwards and Richardson (1968), as shown in Table 2. Change of flow due to adsorption/desorption and reactions, as determined by the overall material balance, is taken into account. The gas is assumed to be an ideal gas.

2. Pressure distribution in the packed-bed adsorptive reactor was described by the mechanical energy equation.

3. The system is nonisothermal. The column wall and the feed stream are maintained at the same constant temperature. Thermal dispersion in the axial direction is considered, with negligible radial gradients. Axial thermal conductivity is estimated using the empirical correlation given by Yagi et al. (1960), as reproduced in Table 2. For a bed packed with spherical particles, the wall-bed heat-transfer coefficient,  $U$ , is given by De Wasch and Froment (1972) and Li and Finlayson (1977). The gas-phase and the catalyst/adsorbent particle are assumed to be in local thermal equilibrium at all times.

4. There exist five components (CH<sub>4</sub>, H<sub>2</sub>, CO<sub>2</sub>, CO, and H<sub>2</sub>O) in an inert carrier (N<sub>2</sub>). The Langmuir model is adopted to describe the adsorption equilibrium for component CO<sub>2</sub>. Hufton et al. (1999) and Ding and Alpay (2000a) reported that steam had a significant effect on the adsorption behavior of CO<sub>2</sub>. In this article, two different sets of Langmuir adsorption parameters are used for wet and dry conditions (Ding and Alpay, 2000a), as shown in Table 2.

5. The linear driving-force (LDF) model is used to represent the mass transport of CO<sub>2</sub> to the adsorbent. Table 2 also shows the correlation to evaluate the value of the LDF mass-transfer coefficient (Ding and Alpay, 2000a).

From the preceding assumptions, we can derive the following governing equations and the corresponding initial conditions and general boundary conditions for the five steps.

Based on Eq. 2a, the actual rates of  $r_i$ ,  $(r_i)_{\text{obv}}$ , ( $i = \text{CH}_4$ , H<sub>2</sub>O, H<sub>2</sub>, CO<sub>2</sub>, CO) can be expressed as

$$(r_i)_{\text{obv}} = \sum_{j=1}^{\text{III}} v_{ij} \eta_j R_j \quad (i = \text{CH}_4, \text{H}_2\text{O}, \text{H}_2, \text{CO}_2, \text{CO}), \quad (3)$$

where  $\eta_j$  is the effectiveness factor for reaction  $j$  ( $j = \text{I}, \text{II}, \text{III}$ ).

Thus, the overall mass balance equation is

$$\epsilon_t \frac{\partial C}{\partial t} + \frac{\partial(uC)}{\partial z} + \rho_{\text{ad}} \frac{\partial \bar{q}_{\text{CO}_2}}{\partial t} - \rho_{\text{cat}} \sum_{i=1}^5 \sum_{j=1}^{\text{III}} v_{ij} \eta_j R_j = 0$$

$$(i = \text{CH}_4, \text{H}_2\text{O}, \text{H}_2, \text{CO}_2, \text{CO}), \quad (4)$$

where  $C$  is the total molar concentration in the bulk phase,  $\epsilon_t$  is the total porosity of the packed bed,  $u$  is the superficial velocity,  $\bar{q}_{\text{CO}_2}$  is the solid-phase concentration for CO<sub>2</sub> (average over an adsorbent particle),  $\rho_{\text{ad}}$  is the mass of adsorbent per bed volume, and  $\rho_{\text{cat}}$  is the mass of catalyst per bed volume. These densities of  $\rho_{\text{ad}}$  and  $\rho_{\text{cat}}$  are related to the bulk packing density  $\rho_b$  by  $\rho_{\text{ad}} = \alpha \rho_b / (1 + \alpha)$ ,  $\rho_{\text{cat}} = \rho_b / (1 + \alpha)$ , where  $\alpha$  is the mass ratio of the adsorbent and catalyst in the packed bed.

The kinetic-energy change is neglected in the mechanical-energy balance; then we have (Sereno and Rodrigues, 1993)

$$\frac{\partial}{\partial t} (\rho_g u) = - \frac{\partial P}{\partial z} - K_D u - K_V u |u|, \quad (5)$$

where  $\rho_g$  is the gas-phase density, and  $K_D$  and  $K_V$  are parameters corresponding to the viscous and kinetic pressure loss terms (see Table 2).

For component  $i$  ( $\bar{q}_i = 0$ , unless for  $i = \text{CO}_2$ ), the mass balance for the packed-bed adsorptive reactor can be written as

$$\epsilon_t \frac{\partial C_i}{\partial t} + \frac{\partial(uC_i)}{\partial z} + \rho_{\text{ad}} \frac{\partial \bar{q}_i}{\partial t} - \rho_{\text{cat}} \sum_{j=1}^{\text{III}} v_{ij} \eta_j R_j = \epsilon_b \frac{\partial}{\partial z} \left( D_L C \frac{\partial y_i}{\partial z} \right)$$

$$(i = \text{CH}_4, \text{H}_2\text{O}, \text{H}_2, \text{CO}_2, \text{CO}), \quad (6)$$

where  $C_i$  is the molar concentration of gas-phase component  $i$  ( $C_i = y_i C$ ),  $\epsilon_b$  is the voidage of the bed, and  $D_L$  is the axial dispersion coefficient.

The LDF model is adopted to describe the mass-transfer rate of CO<sub>2</sub> to the adsorbent

$$\frac{\partial \bar{q}_{\text{CO}_2}}{\partial t} = k_{\text{CO}_2} (q_{\text{CO}_2}^* - \bar{q}_{\text{CO}_2}), \quad (7)$$

where  $k_{\text{CO}_2}$  is the LDF mass-transfer coefficient, and  $q_{\text{CO}_2}^*$  is the equilibrium solid-phase concentration, which are respectively evaluated by the correlation given by Ding and Alpay (2000a) and the Langmuir isotherm (see Table 2).

The energy balance for the bed-volume element, including the heat-transfer to the column wall, is described as (Kikkinides et al., 1993; Ding and Alpay, 2000b)

$$\left[ \epsilon_t C C_{vg} + (\rho_{\text{ad}} + \rho_{\text{cat}}) C_{ps} \right] \frac{\partial T}{\partial t} + C C_{pg} u \frac{\partial T}{\partial z} - \rho_{\text{ad}} \sum_{i=1}^n \left( -\Delta H_{\text{adi}} \frac{\partial \bar{q}_i}{\partial t} \right) - \frac{2U}{R_0} (T_w - T) - \rho_{\text{cat}} \sum_{i=1}^5 \sum_{j=1}^{\text{III}} v_{ij} \eta_j R_j \Delta H_{Rj} = \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right), \quad (8)$$

where  $C_{pg}$  and  $C_{ps}$  are the gas- and solid-phase heat capacity, respectively,  $C_{vg}$  is the heat capacity of the gas phase at constant volume,  $k_z$  is the effective thermal conductivity,  $-\Delta H_{adi}$  is the adsorption heat for component  $i$  (that is,  $\text{CO}_2$ ),  $\Delta H_{Rj}$  is the reaction heat of reaction  $j$ ,  $U$  is the overall bed-wall heat-transfer coefficient, and  $R_0$  is the inner radius of the reactor.

### Initial conditions

At  $t = 0$ ,  $T = T_f$ ,  $u = 0$ ,  $q_i = 0$ ,  $y_{\text{H}_2} = 1$ ,  $y_i = 0$ ,  $P_{\text{H}_2} = P_{\text{H}}$ ,  $P_i = 0$  ( $i = \text{CO}$ ,  $\text{CO}_2$ ,  $\text{H}_2\text{O}$ ,  $\text{CH}_4$ ,  $\text{N}_2$ ).

The final distributions of concentrations, temperature, pressure along the adsorptive reactor column for one step are the initial conditions for the next step.

### General boundary conditions for the five steps

$$\left(\frac{\partial y_i}{\partial z}\right)_{z=0} = \delta_1, \quad \left(\frac{\partial y_i}{\partial z}\right)_{z=L} = \delta_2 \quad (9a)$$

$$\left. \begin{array}{l} (u)_{z=0} \quad (\text{for Steps 1,5}) \\ \left(\frac{\partial u}{\partial z}\right)_{z=0} \quad (\text{for Steps 2-4}) \end{array} \right\} = \delta_3,$$

$$\left. \begin{array}{l} \left(\frac{\partial u}{\partial z}\right)_{z=L} \quad (\text{for Steps 1,5}) \\ (u)_{z=L} \quad (\text{for Steps 2-4}) \end{array} \right\} = \delta_4 \quad (9b)$$

$$\left. \begin{array}{l} (P)_{z=0} \quad (\text{for Steps 1,2}) \\ \left(\frac{\partial P}{\partial z}\right)_{z=0} \quad (\text{for Steps 3-5}) \end{array} \right\} = \delta_5,$$

$$\left. \begin{array}{l} \left(\frac{\partial P}{\partial z}\right)_{z=L} \quad (\text{for Steps 1,2}) \\ (P)_{z=L} \quad (\text{for Steps 3-5}) \end{array} \right\} = \delta_6 \quad (9c)$$

$$\left(\frac{\partial T}{\partial z}\right)_{z=0} = \delta_7, \quad \left(\frac{\partial T}{\partial z}\right)_{z=L} = \delta_8, \quad (9d)$$

in which the expressions of  $\delta$  for the five steps are listed in Table 3.

### Simplification of Governing Equations

Assuming ideal-gas law  $C = P/RT$ , the following equation can be derived from Eq. 6

$$\begin{aligned} \frac{\partial y_i}{\partial t} = & \frac{\epsilon_b}{\epsilon_t} \frac{D_L}{L^2} \left( \frac{\partial^2 y_i}{\partial \zeta^2} + \frac{1}{P} \frac{\partial P}{\partial \zeta} \frac{\partial y_i}{\partial \zeta} - \frac{1}{T} \frac{\partial T}{\partial \zeta} \frac{\partial y_i}{\partial \zeta} \right) \\ & - \frac{1}{\epsilon_t L} \left( u \frac{\partial y_i}{\partial \zeta} + y_i \frac{\partial u}{\partial \zeta} + \frac{u y_i}{P} \frac{\partial P}{\partial \zeta} - \frac{u y_i}{T} \frac{\partial T}{\partial \zeta} \right) \\ & - \frac{\rho_{ad} RT}{\epsilon_t P} \frac{\partial \bar{q}_i}{\partial t} + \frac{\rho_{cat} RT}{\epsilon_t P} \sum_{j=1}^{\text{III}} v_{ij} \eta_j R_j - \frac{y_i}{P} \frac{\partial P}{\partial t} + \frac{y_i}{T} \frac{\partial T}{\partial t}, \quad (10a) \end{aligned}$$

where  $\zeta = z/L$ . The term  $\partial \bar{q}_i / \partial t$  equals zero except for  $i = \text{CO}_2$ ;  $\partial \bar{q}_{\text{CO}_2} / \partial t$  is calculated by Eq. 7.

The flow velocity is derived from the overall material balance (Eq. 4)

$$\begin{aligned} \frac{\partial u}{\partial \zeta} = & \frac{u}{T} \frac{\partial T}{\partial \zeta} - \frac{u}{P} \frac{\partial P}{\partial \zeta} - \frac{\rho_{ad} RTL}{P} \frac{\partial \bar{q}_{\text{CO}_2}}{\partial t} \\ & + \frac{2\rho_{cat} RTL}{P} (\eta_I R_I + \eta_{II} R_{II}) - \frac{\epsilon_t L}{P} \frac{\partial P}{\partial t} + \frac{\epsilon_t L}{T} \frac{\partial T}{\partial t}. \quad (10b) \end{aligned}$$

The interbed pressure dynamics for the variable pressure steps are estimated using a linear dynamic pressure (Serenio and Rodrigues, 1993; Malek and Farooq, 1997)

$$\frac{\partial P}{\partial t} = \begin{cases} 0 & (\text{for step 1}) \\ -(P_H - P_L)/t_2 & (\text{for step 2}) \\ 0 & (\text{for step 3}) \\ 0 & (\text{for step 4}) \\ (P_H - P_L)/t_5 & (\text{for step 5}). \end{cases} \quad (10c)$$

Table 3. Values of  $\delta$  for the Five Steps of the PSAR Cycle

| Step                       | $\delta_1$                                     | $\delta_2$                                    | $\delta_3$ | $\delta_4$ | $\delta_5$ | $\delta_6$ | $\delta_7$                             | $\delta_8$                            |
|----------------------------|--|---|------------|------------|------------|------------|--|---------------------------------------|
| 1. (Reaction + adsorption) | $-\frac{u_{10}(y_{fi} - y_i)}{\epsilon_b D_L}$ | 0   | $u_{10}$   | 0          | $P_H$      | 0          | $-\frac{u_{10} CC_{pg}(T_f - T)}{k_z}$ | 0                                     |
| 2. (Reaction + desorption) | 0  | 0   | 0          | 0          | $P(t)^*$   | 0          | 0                                      | 0                                     |
| 3. Desorption              | 0  | $\frac{u_{30}(y_{fi} - y_i)}{\epsilon_b D_L}$ | 0          | $-u_{30}$  | 0          | $P_L$      | 0                                      | $\frac{u_{30} CC_{pg}(T_f - T)}{k_z}$ |
| 4. Desorption              | 0  | $\frac{u_{40}(y_{fi} - y_i)}{\epsilon_b D_L}$ | 0          | $-u_{40}$  | 0          | $P_L$      | 0                                      | $\frac{u_{40} CC_{pg}(T_f - T)}{k_z}$ |
| 5.                         | 0  | $\frac{u_{50}(y_{fi} - y_i)}{\epsilon_b D_L}$ | 0          | 0          | 0          | $P(t)^*$   | 0                                      | $\frac{u_{50} CC_{pg}(T_f - T)}{k_z}$ |

Note: All velocities  $u_{k0}$  ( $k = 1, 2, \dots, 5$ ) are taken as positive numbers, whatever the flow direction. The index  $k$  stands for the step number and the index 0 stands for the feed side in Steps 1, 3, 4, and 5 and outlet for Step 2.

\*Equation 10c was used (linear ramp).

The mechanical energy balance (Eq. 5) is changed as (Ergun equation)

$$\frac{\partial P}{\partial \zeta} = L(-K_D u - K_V u|u|). \quad (10d)$$

Based on Eq. 8, the energy balance for the bed-volume element that includes the heat-transfer to the column wall is described as

$$\frac{\partial T}{\partial t} = \frac{1}{\left[ \frac{\epsilon_t C_{vg} P}{RT} + (\rho_{ad} + \rho_{cat}) C_{ps} \right]} \times \left[ \frac{k_z}{L^2} \frac{\partial^2 T}{\partial \zeta^2} - \frac{C_{pg} P u}{RTL} \frac{\partial T}{\partial \zeta} + \rho_{ad} (-\Delta H_{ad CO_2}) \frac{\partial \bar{q}_{CO_2}}{\partial t} + 2\rho_{cat} (\eta_I R_I \Delta H_{RI} + \eta_{II} R_{II} \Delta H_{RII}) + \frac{2U}{R_0} (T_w - T) \right]. \quad (10e)$$

Although the preceding model is a simplification of the real situation, it allows us to investigate the whole cyclic process for fine catalyst/adsorbent particles packed in the adsorptive reactor with small diameter.

## Numerical Method

The model equations were numerically solved by the orthogonal collocation method for the continuous hydrogen

production in the PSA reactor. Because of the presence of steep composition gradients within the bed and the periodic reversal in the direction of the gas flow, in this simulation 31 axial collocation points for the packed-bed adsorptive reactor were selected in order to obtain a stable numerical solution.

At all collocation points, Eqs. 10b and 10d are discretized into a set of linear algebraic equations that are solved numerically by the Gauss method in order to obtain the velocity and pressure distributions along the adsorptive reactor. Equations 10a and 10e are discretized in the axial direction, leading to a set of ordinary differential equations that are integrated in the time domain using Gear's stiff variable step integration routine in order to obtain the effluent mole fraction and temperature histories and the concentration and temperature profiles along the adsorptive reactor.

The sequence for solving numerically the PSA reactor at each step is as follows:

1. Calculate the reaction rates using Eqs. 1a, 1b, 1c
2. Calculate the pressure derivative using the linear dynamic model Eq. 10c
3. Calculate the adsorptive loading derivative by Eq. 7
4. Solve Eq. 10b for the velocity
5. Solve Eq. 10d for the pressure
6. Solve Eqs. 9a and 9d for the boundary conditions
7. Solve Eq. 10a for the species mole fraction derivative
8. Solve Eq. 10e for the temperature derivative.

At the end of each step, the gas- and solid-phase component concentrations, temperature, and pressure were stored

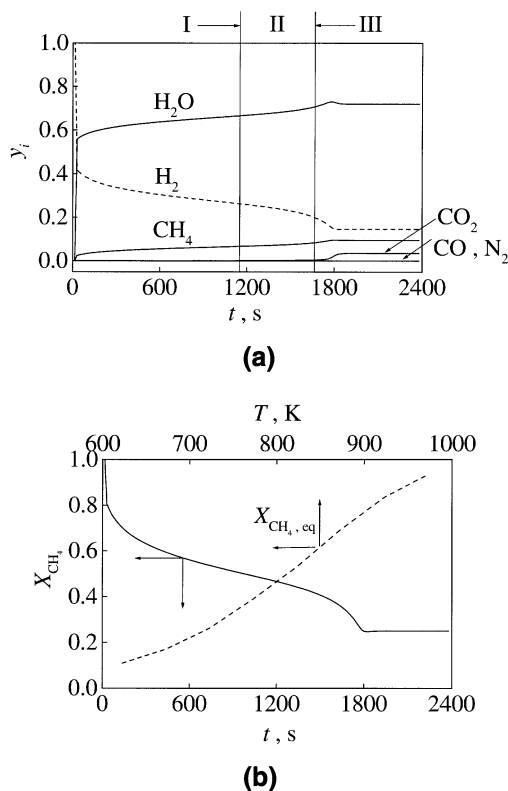
**Table 4. Reference Parameters Values Used in the Simulations for Step 1**

| Parameters (Constants)         | Values   |                    |
|--------------------------------|--|--------------------|
| $b_{CO_2}^*$                   | $2.36 \times 10^{-4} \text{ Pa}^{-1}$ (wet condition)<br>$1.69 \times 10^{-4} \text{ Pa}^{-1}$ (dry condition) | Change with $T$    |
| $C_{pg}^*$                     | 42 J/mol·K   | Constant           |
| $C_{ps}^*$                     | 850 J/kg·K   | Constant           |
| $d_p^*$                        | $5 \times 10^{-4} \text{ m}$   | Constant           |
| $D_L^{**}$                     | $1.6 \times 10^{-5} \text{ m}^2/\text{s}$  | Change with $u$    |
| $R_0^*$                        | 0.0125 m   | Constant           |
| $K_D^{**}$                     | $42,103 \text{ N} \cdot \text{s}/\text{m}^4$   | Constant           |
| $K_V^{**}$                     | $21,966 \text{ N} \cdot \text{s}^2/\text{m}^5$   | Change with $T, P$ |
| $k_z^*$                        | $0.29 \text{ J}/\text{m} \cdot \text{s} \cdot \text{K}$  | Change with $u$    |
| $\Delta H_{ad CO_2}^*$         | $-17,000 \text{ J/mol}$  | Constant           |
| $m_{CO_2}^*$                   | 0.65 mol/kg (wet condition)<br>0.63 mol/kg (dry condition)   | Constant           |
| $P_H^*$                        | 445.7 kPa  | Constant           |
| $P_L^\dagger$                  | 125.7 kPa  | Constant           |
| $T_f^*$                        | 723 K  | Constant           |
| $T_w^\dagger$                  | 723 K  | Constant           |
| $U^{**}$                       | $71 \text{ J}/\text{m}^2 \cdot \text{K}$   | Change with $u$    |
| $\epsilon_b^*$                 | 0.48   | Constant           |
| $\epsilon_p^*$ (for adsorbent) | 0.24   | Constant           |
| $\epsilon_t^*$                 | 0.64   | Constant           |
| $\mu^*$                        | $2.87 \times 10^{-5} \text{ Pa} \cdot \text{s}$  | Constant           |
| $\rho_{ad}^\dagger$            | 498 kg/m <sup>3</sup>  | Constant           |
| $\rho_{cat}^\dagger$           | 249 kg/m <sup>3</sup>  | Constant           |
| $\eta_j^*$                     | 1.0  | Constant           |

\*Data from Ding and Alpay (2000a,b).

\*\*Data evaluated from Table 2.

†Present work.



**Figure 2. (a) Typical effluent mole fraction  $y_i$  and (b) methane conversion  $X_{CH_4}$  vs. time  $t$  for Step 1.**

$P_H = 445.7$  kPa;  $T_f = T_w = 450^\circ\text{C}$ ;  $L = 2$  m;  $u_{10} = 0.05$  m/s;  $H_2O/CH_4 = 6$ ;  $\rho_{ad} = 498$  kg/m<sup>3</sup>;  $\rho_{cat} = 249$  kg/m<sup>3</sup>;  $m_{CO_2} = 0.65$  mol/kg;  $b_{CO_2} = 2.36 \times 10^{-4}$  Pa<sup>-1</sup>.

and used as the initial conditions for the next step in one cycle.

The adsorption behavior of  $CO_2$  onto the adsorbent under wet and dry conditions is different (Hufton et al., 1999; Ding and Alpay, 2000a); therefore, we take the data under wet conditions for Steps 1 and 2, and the data under dry condition for Steps 3 to 5.

The criteria to switch from one step to the next are: the average purity of hydrogen product for Step 1 equals 80%, and the exit mole fraction of  $CO_2$  for Step 3 must be lower than  $1.0 \times 10^{-2}$ . The time duration of Steps 2, 4, and 5 were all set in the 100s. The criterion for cyclic steady-state operation was based on the average purity of the hydrogen product.

## Results and Discussion

### Adsorption-enhanced steam–methane reforming process

Figure 2 shows typical effluent mole fraction,  $y_i$  (Figure 2a), and methane conversion (Figure 2b) as a function of time,  $t$ , for the adsorption-enhanced reaction process at  $P_H = 445.7$  kPa,  $L = 2$  m,  $T_f = T_w = 450^\circ\text{C}$ ,  $u_{10} = 0.05$  m/s,  $H_2O/CH_4 = 6$  (the mole fraction of inert gas is about 0.001), and the mass ratio of adsorbent to catalyst = 2; the other reference data are listed in Table 4.

The basic parameters for the simulation of the adsorption-enhanced reaction step are selected from Ding and Alpay (2000a,b). The diameters of the adsorbent and catalyst,  $d_p$ , are chosen as 0.5 mm. To calculate the effectiveness factors, we should consider the catalyst and its properties in more detail. For our simulation system, the catalyst is uniform and small in size; for convenience, we chose  $\eta_j = 1.0$  and assumed that  $\eta_j$  is the same for different reactions. The properties of the gas mixture change with pressure, temperature, and other operating conditions; the reference values for Step 1 are listed in Table 4.

In Figure 2a, it is evident that the whole region can be divided into three zones according to the adsorption behavior of  $CO_2$ . (I) The adsorption-enhanced reaction zone, where the effluent mole fraction of  $CO_2$  is lower (mole fraction is about  $9.4 \times 10^{-3}$ ), adsorption enhances the conversion of methane, and the effluent mole fraction of hydrogen at the exit is higher; at the same time, the coproduction of CO is suppressed (Hufton et al., 1999). (II) The breakthrough zone of  $CO_2$ , where the mole fraction of  $CO_2$  increases from  $9.4 \times 10^{-3}$  to  $3.8 \times 10^{-2}$ , the adsorbent is nearly saturated by  $CO_2$  in the operation zone of the reactor, the enhancement is not evident, and the purity of hydrogen at the effluent gas drops rapidly. (III) The equilibrium state zone, where the adsorbent is saturated almost entirely by  $CO_2$ , the operation mode is the same as the conventional one, that is, the steam–methane reforming reaches the steady state. At the steady state, the conversion of methane reaches 24.5%, which is in reasonable agreement with the experimental data of Ding and Alpay (2000b) (24.0%) and Hufton et al. (1999) (28.0%). At the initial state of the adsorption-enhanced reaction zone (for 0 ~ 300 s), methane conversion is larger than 65%, while for conventional reactor (without adsorbent), the reaction temperature should be controlled above  $600^\circ\text{C}$  in order to reach the same conversion (Figure 2b):

$y_i$  (dry basis) and the conversion of methane  $X_{CH_4}$  are respectively defined as

$$y_i \text{ (dry basis)} = \frac{y_i}{1 - y_{H_2O}} \quad (11a)$$

$$X_{CH_4} = \frac{\text{Feed of } CH_4 \text{ (mol/s)} - \text{effluent of } CH_4 \text{ (mol/s)}}{\text{Feed of } CH_4 \text{ (mol/s)}} \\ = 1 - \left( \frac{RT_f}{u_{10} P_H y_{0CH_4}} \right)_{\text{feed}} \left( \frac{u P y_{CH_4}}{RT} \right)_{\text{outlet}} \quad (11b)$$

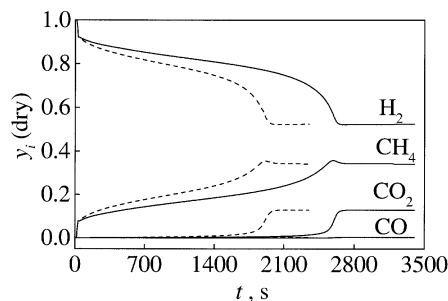
At the equilibrium state (without adsorbent), we can derive the following relationship to check the simulation results

$$X_{eq, CH_4} = \frac{1 - y_{eq, CH_4} \text{ (dry basis)}}{1 + 4 y_{eq, CH_4} \text{ (dry basis)}}, \quad (11c)$$

that is, at the equilibrium state, the result from Eq. 11b should be equal to that from Eq. 11c.

The adsorbent, which can adsorb  $CO_2$  at high temperature, is usually made of  $K_2CO_3$ –hydrotalcite (Hufton et al., 1999). At  $450^\circ\text{C}$ , the saturated  $CO_2$  adsorption capacity is

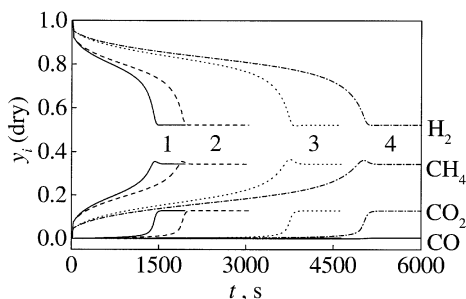




**Figure 3. Effect of adsorbent capacity for CO<sub>2</sub> on H<sub>2</sub> purity of effluent gas vs. time for Step 1.**

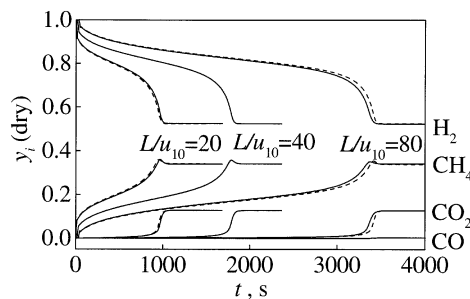
Common conditions:  $P_H = 445.7$  kPa;  $T_f = T_w = 450^\circ\text{C}$ ;  $L = 2$  m;  $u_{10} = 0.05$  m/s;  $\text{H}_2\text{O}/\text{CH}_4 = 6$ ;  $\rho_{\text{ad}} = 498$  kg/m<sup>3</sup>;  $\rho_{\text{cat}} = 249$  kg/m<sup>3</sup>. Solid line:  $m_{\text{CO}_2} = 0.85$  mol/kg;  $b_{\text{CO}_2} = 3.94 \times 10^{-4}$  Pa<sup>-1</sup> (Hufton et al., 1999). Dashed line:  $m_{\text{CO}_2} = 0.65$  mol/kg;  $b_{\text{CO}_2} = 2.36 \times 10^{-4}$  Pa<sup>-1</sup> (Ding and Alpay, 2000b).

about 0.5 ~ 1.0 mol/kg. Figure 3 shows the effect of the CO<sub>2</sub> adsorption capacity on the effluent concentrations from the adsorptive reactor, where the solid lines were calculated with  $m_{\text{CO}_2} = 0.85$  mol/kg,  $b_{\text{CO}_2} = 3.94 \times 10^{-4}$  Pa<sup>-1</sup> (Hufton et al., 1999), and the dashed lines with  $m_{\text{CO}_2} = 0.65$  mol/kg,  $b_{\text{CO}_2} = 2.36 \times 10^{-4}$  Pa<sup>-1</sup> (Ding and Alpay, 2000a). It is evident that the adsorbent with higher CO<sub>2</sub> adsorption capacity will lead to higher methane conversion and higher hydrogen purity in the product gas. However, it is difficult to improve the adsorption capacity of the adsorbent at high temperature; it is better to adjust the packing density of the adsorbent and the catalyst to enhance the methane conversion. In Figure 4, a comparison is made between the performance of the adsorptive reactor under two different packing densities, that is, between the data of Ding and Alpay (2000a) and that of Hufton et al. (1999). The isotherm parameters use the relevant ones, that is, those of Ding and Alpay (2000a) for simulations with a bulk packing density of 747 kg/m<sup>3</sup>, and those of Hufton et al. (1999) when using a bulk packing density of 1,538 kg/m<sup>3</sup>. Figure 4 shows that the performance improvement could be obtained by the use of a superior adsorbent



**Figure 4. Effect of density of adsorbent and catalyst on H<sub>2</sub> purity of effluent gas vs. time  $t$  for Step 1.**

Common conditions:  $P_H = 445.7$  kPa;  $T_f = T_w = 450^\circ\text{C}$ ;  $L = 2$  m;  $u_{10} = 0.05$  m/s;  $\text{H}_2\text{O}/\text{CH}_4 = 6$ . Line 1 ( $\rho_{\text{ad}}/\rho_{\text{cat}} = 1$ ; solid line) and line 2 ( $\rho_{\text{ad}}/\rho_{\text{cat}} = 2$ ; dashed line): bulk packing density 747 kg/m<sup>3</sup>, with  $m_{\text{CO}_2} = 0.65$  mol/kg and  $b_{\text{CO}_2} = 2.36 \times 10^{-4}$  Pa<sup>-1</sup> (Ding and Alpay, 2000b). Line 3 ( $\rho_{\text{ad}}/\rho_{\text{cat}} = 1$ ; dotted line) and line 4 ( $\rho_{\text{ad}}/\rho_{\text{cat}} = 2$ ; dashed-dot line): bulk packing density 1,538 kg/m<sup>3</sup> with  $m_{\text{CO}_2} = 0.85$  mol/kg and  $b_{\text{CO}_2} = 3.94 \times 10^{-4}$  Pa<sup>-1</sup> (Hufton et al., 1999).



**Figure 5. Effect of residence time on H<sub>2</sub> purity vs. time for Step 1.**

Common conditions:  $P_H = 445.7$  kPa;  $T_f = T_w = 450^\circ\text{C}$ ;  $\text{H}_2\text{O}/\text{CH}_4 = 6$ ;  $\rho_{\text{ad}} = 498$  kg/m<sup>3</sup>;  $\rho_{\text{cat}} = 249$  kg/m<sup>3</sup>;  $m_{\text{CO}_2} = 0.65$  mol/kg;  $b_{\text{CO}_2} = 2.36 \times 10^{-4}$  Pa<sup>-1</sup>. Solid line:  $L = 2$  m with change of feed velocity  $u_{10}$ ; dashed line:  $u_{10} = 0.05$  m/s with change of length  $L$ .

[lines 3 and 4: bulk packing density 1,538 kg/m<sup>3</sup> with  $m_{\text{CO}_2} = 0.85$  mol/kg,  $b_{\text{CO}_2} = 3.94 \times 10^{-4}$  Pa<sup>-1</sup> for Hufton et al. (1999); lines 1 and 2: bulk packing density 747 kg/m<sup>3</sup> with  $m_{\text{CO}_2} = 0.65$  mol/kg,  $b_{\text{CO}_2} = 2.36 \times 10^{-4}$  Pa<sup>-1</sup> for Ding and Alpay (2000a)].

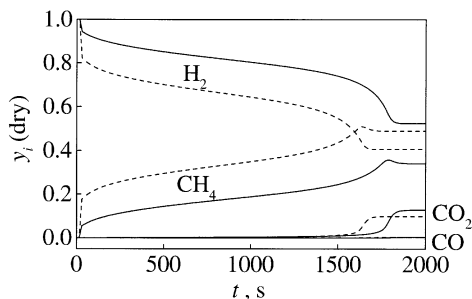
In addition, controlling the residence time ( $L/u_{10}$ ) of the feedgas in the adsorptive reactor will also be a good method for improving the H<sub>2</sub> purity in the effluent gas and the productivity. Increasing the residence time of the feed gas passing through the adsorptive reactor by increasing the length,  $L$ , or by decreasing the feed velocity,  $u_{10}$ , leads to a higher purity of the H<sub>2</sub> product gas, and better productivity can be obtained during the adsorption-enhanced reaction process, as shown in Figure 5. The S/C (steam/methane) ratio is also a key parameter; a higher S/C ratio will be favorable for obtaining higher purity H<sub>2</sub>, as shown in Figure 6.

For a given system (specified the adsorbent and the temperature), increasing the packing density, the adsorbent/catalyst ratio, and the residence time favors the production of high-purity H<sub>2</sub>.

Hufton et al. (1999) carried out experiments for the production of hydrogen by the sorption-enhanced steam-methane reforming process, where a long reactor with higher packing adsorbent and catalyst densities was selected (see Table 5). Higher purity of H<sub>2</sub> is obtained due to small feed velocity and adsorbent with higher CO<sub>2</sub> adsorption capacity. When the net H<sub>2</sub> productivity is 0.8 mol/kg (solid), the purity of the H<sub>2</sub> product is 96%. The simulation results based on their experimental conditions are shown in Figure 7. If controlled, the reaction time is 995 s, the same net H<sub>2</sub> productivity (that is, 0.8 mol/kg of the solid) can be obtained, and the H<sub>2</sub> product purity is 92%.

Here, the average purity of hydrogen (dry basis),  $y_{\text{H}_2}$  (av), at the given operating time,  $t_1$ , is calculated by

$$y_{\text{H}_2}(\text{av}) = \frac{1}{\int_0^{t_1} \left( \frac{Pu(1-y_{\text{H}_2\text{O}})}{RT} \right)_{\text{outlet}} dt} \int_0^{t_1} \left( \frac{Puy_{\text{H}_2}}{RT} \right)_{\text{outlet}} dt. \quad (11d)$$



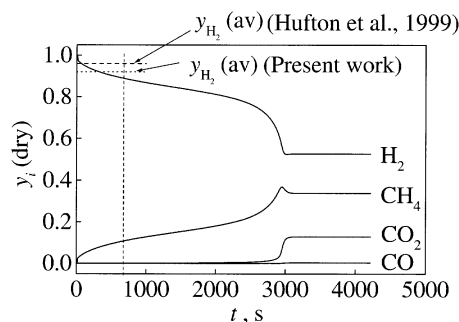
**Figure 6. Effect of mole ratio of steam to methane on  $H_2$  purity of effluent gas vs. time for Step 1.**

Common conditions:  $P_H = 445.7$  kPa;  $T_f = T_w = 450^\circ\text{C}$ ;  $L = 2$  m;  $u_{10} = 0.05$  m/s;  $m_{CO_2} = 0.65$  mol/kg;  $b_{CO_2} = 2.36 \times 10^{-4}$  Pa $^{-1}$ . Solid line:  $H_2O/CH_4 = 6$ ; dashed line:  $H_2O/CH_4 = 3$ .

Ding and Alpay (2000b) also carried out experiments. Figure 8 shows a comparison of our simulation results with their experimental data (for the experimental conditions, see Table 5). The prediction is good except in the initial period; the deviation from the experimental data in the initial period is partly due to the lack of the adsorption data of  $CO_2$  at relatively low concentrations. For their experimental conditions, the  $H_2$  purity in the effluent gas is lower due to the shorter residence time of feed gas in the adsorptive reactor.

#### Continuous production of $H_2$ under cyclic steady state

For industrialization, the high purity of  $H_2$  product gas should be continuously produced from the adsorptive reactor under cyclic steady state; here a five-step one-bed cyclic process is adopted. In this cyclic process, the adsorption-enhanced steam-methane reforming is performed at high pressure ( $P_H = 445.7$  kPa) and the regeneration of the adsorbent that was saturated with  $CO_2$  is carried out at low pressure ( $P_L = 125.7$  kPa), where the adsorptive reactor is purged first with methane, and then with part of the hydrogen product gas. In the simulation, the adsorptive reactor length is 2 m with a diameter of 25 mm, which is packed with the catalyst and adsorbent mixture (the densities of the catalyst and adsorbent are 249 kg/m $^3$  and 498 kg/m $^3$ , respectively). The ad-



**Figure 7. Effluent mole fraction  $y_i$  vs. time based on experimental data of Hufton et al. (1999) for Step 1.**

$P_H = 445.7$  kPa;  $T_f = T_w = 450^\circ\text{C}$ ;  $L = 1.067$  m;  $u_{10} = 0.037$  m/s;  $H_2O/CH_4 = 6$ ;  $\rho_{ad} = 769$  kg/m $^3$ ;  $\rho_{cat} = 769$  kg/m $^3$ ;  $m_{CO_2} = 0.85$  mol/kg;  $b_{CO_2} = 3.94 \times 10^{-4}$  Pa $^{-1}$ .

sorption equilibrium uses data from Ding and Alpay (2000a). It is assumed that the linear change of the pressure occurs with time in the depressurization step and the pressurization step (Eq. 10c).

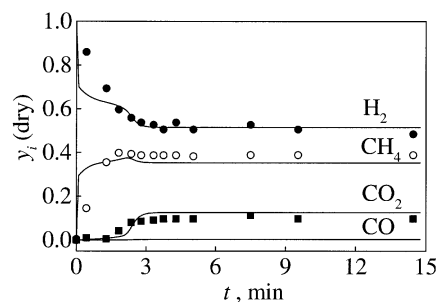
It should be noted that the adsorptive reactor is saturated with  $H_2$  only at Step 1 of the first cycle.

Figure 9 shows the changes in pressure, temperature, and the flow velocity at  $z = 0$  m, in the middle of the bed at  $z = 1$  m, and at the other extremity of the bed ( $z = 2$  m) during one cycle. We take the 15th cycle, which is at the cyclic steady state for the continuous production of hydrogen, as an example. The negative velocity means that the flow is in the countercurrent direction to that of the feed gas flow. For Step 1, the feed velocity is  $u_{10} = 0.05$  m/s, the operation time is controlled at  $t_1 = 1,000$  s in order to obtain greater than 80% average purity of hydrogen in the effluent gas mixture (impure  $H_2$  50–90% is acceptable with fuel cell applications as long as the CO level is kept below 30 ppm) (Hufton et al., 2000). The countercurrent depressurization time is set to be  $t_2 = 100$  s, the countercurrent purge time with methane is controlled at  $t_3 = 900$  s based on the exit mole fraction of  $CO_2$ , and the countercurrent purge time ( $t_4$ ) with part of the

**Table 5. Experimental Conditions from the Literature**

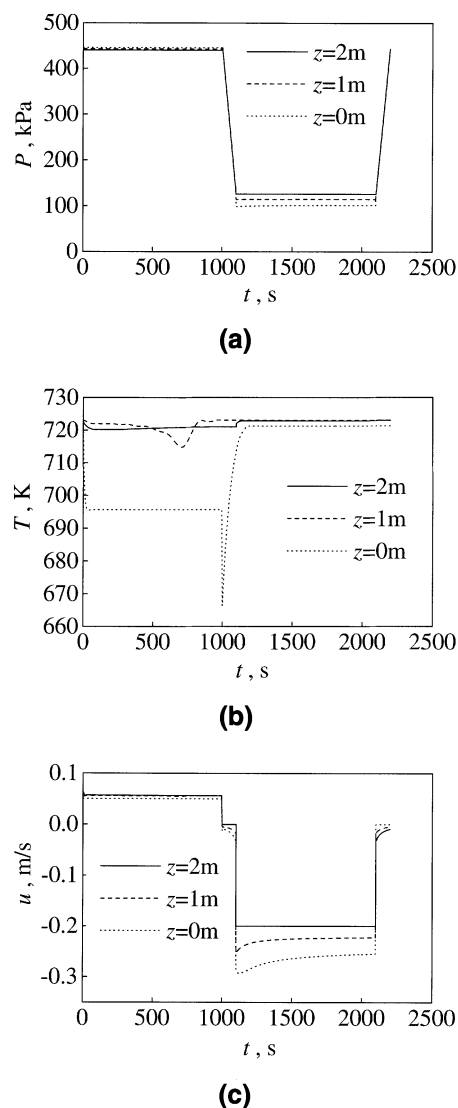
|  | Ding and Alpay<br>(2000a,b) | Hufton et al.<br>(1999) |
|--|-----------------------------|-------------------------|
| Mole ratio $H_2O/CH_4$   | 6                           | 6                       |
| Reactor length $L$ (m)   | 0.22                        | 1.067                   |
| ID of reactor $2R_0$ (mm)                                      | 12.7                        | 38                      |
| Dia. of particles $d_p$ (mm)                                   | 0.5                         | 3                       |
| Effectiveness factor $\eta_j$                                  | 1.0                         | 0.8*                    |
| Superficial velocity $u_{10}$ (m/s)                            | 0.13                        | 0.037                   |
| Dens. of adsorbent $\rho_{ad}$ (kg/m $^3$ ) in<br>ads. reactor | 609                         | 769                     |
| Dens. of catalyst $\rho_{cat}$ (kg/m $^3$ ) in<br>ads. reactor | 139                         | 769                     |
| Parameter for adsorption equil.<br>$m_{CO_2}$ (mol/kg)         | 0.65                        | 0.85                    |
| Parameter for adsorption equil.<br>$b_{CO_2}$ (Pa $^{-1}$ )    | $2.36 \times 10^{-4}$       | $3.94 \times 10^{-4}$   |

\* Xiu et al. (2002).



**Figure 8. Comparison of simulated results with experimental data of Ding and Alpay (2000b) for Step 1.**

$P_H = 445.7$  kPa;  $T_f = 450^\circ\text{C}$ ;  $T_w = 457^\circ\text{C}$ ;  $L = 0.22$  m;  $u_{10} = 0.13$  m/s;  $H_2O/CH_4 = 6$ ;  $\rho_{ad} = 609$  kg/m $^3$ ;  $\rho_{cat} = 139$  kg/m $^3$ ;  $m_{CO_2} = 0.65$  mol/kg;  $b_{CO_2} = 2.36 \times 10^{-4}$  Pa $^{-1}$ . Solid line: simulated results; points: experimental data (Ding and Alpay, 2000b).



**Figure 9. (a) Pressure, (b) temperature, and (c) velocity histories in the adsorptive reactor for the 15th cycle at various axial positions.**

Adsorptive reactor conditions:  $P_H = 445.7$  kPa;  $T_f = T_w = 450^\circ\text{C}$ ;  $L = 2$  m;  $\rho_{\text{cat}} = 249$  kg/m<sup>3</sup>;  $\rho_{\text{ad}} = 498$  kg/m<sup>3</sup> with  $m_{\text{CO}_2} = 0.65$  mol/kg;  $b_{\text{CO}_2} = 2.36 \times 10^{-4}$  Pa<sup>-1</sup>. Cyclic operation conditions:  $P_H = 445.7$  kPa;  $P_L = 125.7$  kPa;  $T_f = T_w = 450^\circ\text{C}$ ;  $u_{10} = 0.05$  m/s;  $u_{30} = u_{40} = 0.2$  m/s;  $t_1 = 1,000$  s;  $t_3 = 900$  s;  $t_2 = t_4 = t_5 = 100$  s;  $\text{H}_2\text{O}/\text{CH}_4 = 6$ .

hydrogen product gas and the countercurrent pressurization time ( $t_5$ ) are set to be 100 s, respectively.

As shown in Figure 9a, for the high-pressure adsorption/reaction step (Step 1) and the low-pressure purge steps (Steps 3 and 4), the pressure drop along the adsorptive reactor is higher for a higher gas velocity. It is 5.7 kPa with 0.05 m/s feed gas velocity for Step 1 and 24.8 kPa with 0.2 m/s purge gas velocity for Steps 3 and 4.

$$\text{mol H}_2/\text{g (solid)} = \frac{\int_0^{t_1} \left( \frac{P u y_{\text{H}_2}}{RT} \right)_{\text{Outlet step 1}} \text{Adt} - \left( \frac{P_H u_{40} y_{\text{H}_2} A t_4}{RT} \right)_{\text{Input step 4}} - \int_0^{t_5} \left( \frac{P u_{50} y_{\text{H}_2}}{RT} \right)_{\text{Input step 5}} \text{Adt}}{AL(\rho_{\text{ad}} + \rho_{\text{cat}})} = 0.63 \text{ mol/kg (solid)}. \quad (13)$$

The temperature change, which is caused by the endothermic reaction and adsorption, will be noticed in the high pressure adsorption/reaction step (Step 1) and the depressurization step (Step 2), as shown in Figure 9b. However, in the purge step (Step 3), the temperature change along the adsorptive reactor is observed only at the initial purge stage (about 200 s) due to the lower rate of CO<sub>2</sub> desorption from the adsorbent.

The number of moles increases for the steam-methane reforming reaction, therefore the output velocity from the reactor ( $z = 2$  m) is higher than the feed velocity ( $z = 0$  m) in the high-pressure adsorption/reaction step (Step 1), as shown in Figure 9c. At the initial purge stage, the change in output velocity ( $z = 0$  m) is significant due to the desorption of CO<sub>2</sub> from the adsorbent, then the change is slow, and the output velocity is higher than the input velocity ( $z = 2$  m). In the depressurization step, the output velocity ( $z = 0$  m) increases with time, while in the pressurization step, the input velocity ( $z = 2$  m) decreases with time.

Figure 10 shows the temperature, pressure, and velocity distributions along the adsorptive reactor at the end of the high pressure adsorption/reaction step (Step 1; Figure 10a), the countercurrent blowdown step (Step 2; Figure 10b), the purge step (Steps 3 and 4; Figure 10c), and the repressurization step (Step 5; Figure 10d).

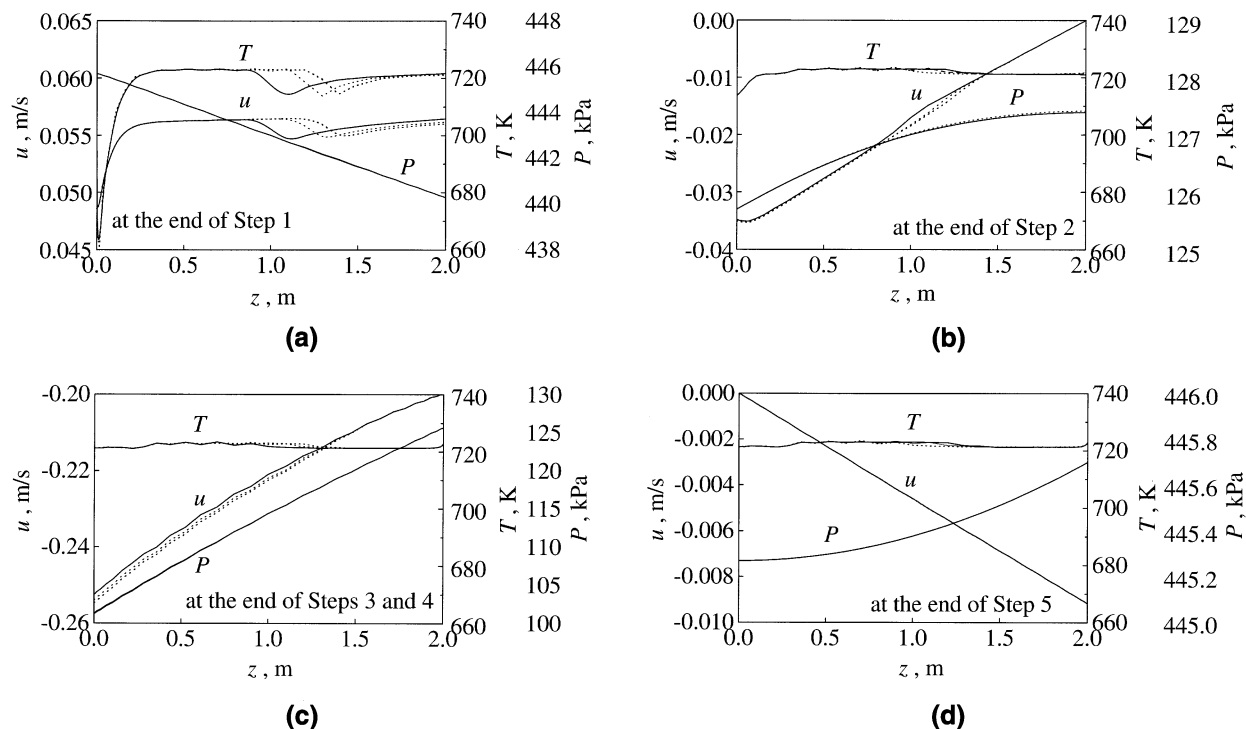
Figure 11a shows the effluent mole fractions,  $y_i$  (dry), for the first cycle (solid lines), the 2nd, 5th, and 15th cycles (dotted lines) from the adsorptive reactor in Step 1 during the continuous production of H<sub>2</sub> under the operating conditions just mentioned. The purity of H<sub>2</sub> in the product gas (dry basis) in the first cycle is higher than that in the next cycles since the first cycle uses the fresh adsorbent. After regeneration, the adsorption capacity of the adsorbent will decrease slightly due to the remaining CO<sub>2</sub> in the adsorbent; therefore, the purity of the hydrogen will decrease. Figure 11b shows the average purity of the H<sub>2</sub> in the product gas obtained from Step 1 during the continuous production process as a function of the cycle number  $n$ . The average purity of the H<sub>2</sub> in the product gas (dry basis),  $y_{\text{H}_2}(\text{av})$ , will decrease with the cycle number. However, the cyclic steady state is reached quickly in our simulation conditions, as shown in Figure 11b; 15 cycles are sufficient to reach the cyclic steady state for the following criterion when  $n \geq 15$ :

$$\left| \frac{y_{\text{H}_2}^{(n)}(\text{av}) - y_{\text{H}_2}^{(n-1)}(\text{av})}{y_{\text{H}_2}^{(n)}(\text{av})} \right| \leq 1.0 \times 10^{-4}, \quad (12)$$

where  $n$  is the number of the cycle.

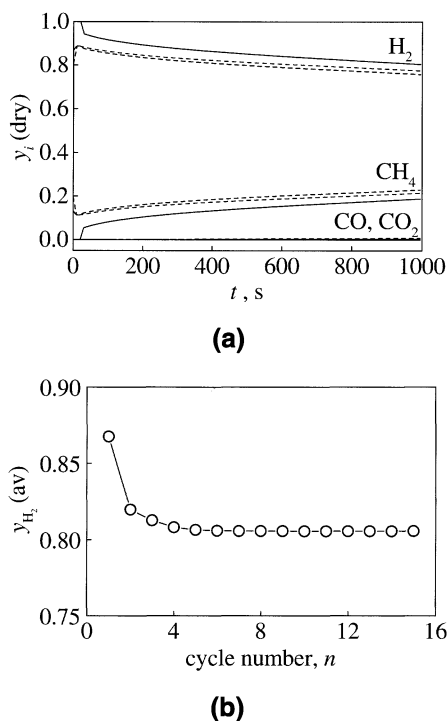
At the cyclic steady state, hydrogen gas with an average purity of over 80%, with traces of the CO and CO<sub>2</sub>, can be continuously produced; the productivity of hydrogen is about 0.63 mol/kg (solid) in one cycle.

Here, the productivity of H<sub>2</sub> per gram of solid in one cycle is calculated by



**Figure 10. Pressure, temperature, and velocity axial distributions in cyclic process.**

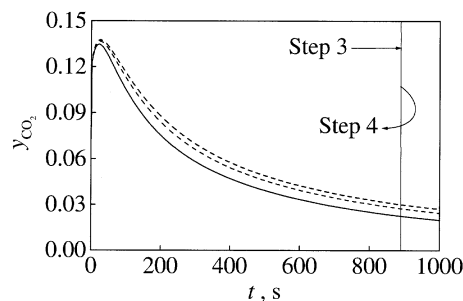
(a) At the end of Step 1; (b) at the end of Step 2; (c) at the end of Steps 3 and 4; (d) at the end of Step 5. The adsorptive reactor conditions and the cyclic operation conditions are the same as in Figure 9. Solid line: 1st cycle; dotted line: 2nd, 5th, 15th cycles.



**Figure 11. (a) Effluent mole fraction  $y_i$  vs. time  $t$  in Step 1 for the 1st, 2nd, 5th, and 15th cycles; (b) and average purity of  $H_2$  in product gas vs. cycle number  $n$ .**

The adsorptive reactor conditions and the cyclic operation conditions are the same as in Figure 9. (a) Solid line: 1st cycle; dotted line: 2nd, 5th, 15th cycles.

The regeneration of the adsorbent that is saturated with  $CO_2$  in Step 1 should be a key step for the continuous production of high-purity hydrogen. If the regeneration is not enough, it is impossible to continuously produce high-purity hydrogen in the next cycle. We purge the adsorptive reactor with methane at a lower pressure to desorb  $CO_2$  from the adsorbent (Step 3), and only a small fraction of hydrogen product gas is used to remove the remaining methane from the interparticle space of the reactor near the end of the purge step (Step 4). Figure 12 shows the  $CO_2$  effluent mole fraction vs. time during the two purge steps. It is obvious that only in the initial period of the purge (0 ~ 400 s) is the  $CO_2$  desorption rate faster; after 600 s the desorption rate is slow, which means a longer purge time is needed.

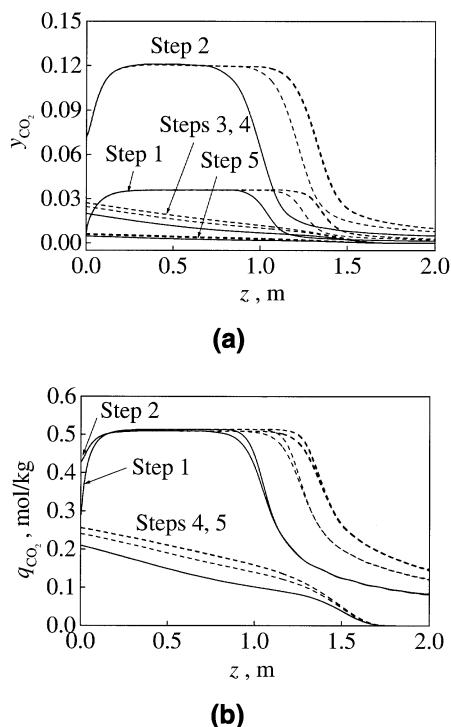


**Figure 12. Effluent  $CO_2$  mole fraction vs. time in Steps 3 and 4 in cyclic process.**

The adsorptive reactor conditions and the cyclic operation conditions are the same as in Figure 9. Solid line: 1st cycle; dotted line: 2nd, 5th, and 15th cycles.

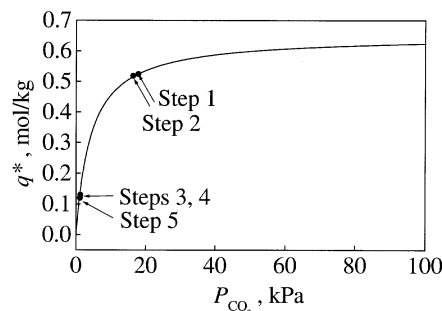
Carvill et al. (1996) pointed out that at least two parallel reactors are needed to run the cycle in a continuous mode. The cycle times for the steps are designed so that one reactor undergoes Step 1 for a period of time while the companion reactor undergoes Steps 2–5 during the same period of time. Our simulation results could not meet this condition, since  $t_1 = 1,000$  s and  $\sum_{i=2}^5 t_i = 1,200$  s. The reason is that regeneration of the adsorbent is difficult; increasing the velocity of the carrying gas will favor the regeneration process, but lead to a higher pressure drop along the adsorptive reactor. Therefore, it is necessary to find a high-efficiency regeneration method. It should be noted that our process could be operated continuously by using two beds: when one bed is in Steps 5 and 1 ( $t_1 + t_5 = 1,100$  s), the other is in Steps 2, 3, and 4 ( $t_2 + t_3 + t_4 = 1,100$  s).

Figures 13a and 13b show  $\text{CO}_2$  mole fraction distributions in the gas- and solid-phases along the adsorptive reactor at the end of each step. After the high-pressure adsorption/reaction step (Step 1), the adsorption front for  $\text{CO}_2$  is located at about  $z = 1.1$  m, as shown in Figure 13b, that is, the adsorbent between  $z = 0$  and 1.1 m is saturated with  $\text{CO}_2$ , so the length of the unused bed (LUB) of the packed bed is  $(2 - 1.1)/2 = 0.45$ . During the countercurrent depressurization step (Step 2), the reversible reactions and  $\text{CO}_2$  desorption take place simultaneously, the mole fraction of  $\text{CO}_2$  in the gas-phase increases rapidly (Figure 13a), but the concentration of  $\text{CO}_2$  in the solid-phase decreases slightly, and the



**Figure 13. (a) Mole fraction of  $\text{CO}_2$  in the gas phase vs.  $z$ , and (b)  $\text{CO}_2$  adsorbent concentration vs. axial distance of reactor at the end of each step in the cyclic process.**

The adsorptive reactor conditions and the cyclic operation conditions are the same as in Figure 9. Solid line: 1st cycle; dashed line: 2nd, 5th, 15th cycles.



**Figure 14. Operating points of partial pressure of  $\text{CO}_2$  at the end of steps for one cycle: the adsorptive reactor conditions and the cyclic operation conditions are the same as in Figure 9.**

distribution of  $\text{CO}_2$  spreads (Figure 13b). In the purge step, the first purge is with methane (Step 3), and the second purge with a part of the hydrogen product gas (Step 4) to replace the remaining methane in the interparticle space of the adsorptive reactor. With two purge steps, the  $\text{CO}_2$  in the adsorbent is desorbed and taken away by the inert gas; the adsorbent is regenerated and the  $\text{CO}_2$  concentration in the adsorbent drops to an acceptable level, as shown in Figures 13a and 13b.

In the last step of one cycle, that is, the pressurization step (Step 5), the adsorptive reactor is pressurized with part of the hydrogen product gas to a high pressure,  $P_H$ . Increasing pressure always favors the adsorption of  $\text{CO}_2$ ; therefore, the  $\text{CO}_2$  concentration in the gas-phase will decrease after the pressurization step, as shown in Figure 13a, but the  $\text{CO}_2$  concentration in the adsorbent decreases only slightly (Figure 13b). After the pressurization step, the next cycle will start.

Figure 14 shows that Step 2 (countercurrent depressurization from  $P_H = 445.7$  kPa to  $P_L = 125.7$  kPa, operating time  $t_2 = 100$  s) makes a minor contribution to the desorption of  $\text{CO}_2$ , because the partial pressures of  $\text{CO}_2$  (about 17.8 kPa at the end of Step 1,  $y_{\text{CO}_2} \approx 0.04$ , and 16.3 kPa at the end of Step 2,  $y_{\text{CO}_2} \approx 0.13$ ) are in the saturated region of the  $\text{CO}_2$  isotherm. After Steps 3 and 4 (operating pressure  $P_L = 125.7$  kPa, operating time  $t_3 + t_4 = 1,000$  s, first purge with methane for  $t_3 = 900$  s, purge velocity  $u_{30} = 0.2$  m/s, then purge with hydrogen for  $t_4 = 100$  s, purge velocity  $u_{40} = 0.2$  m/s), the mole fraction of  $\text{CO}_2$  in the gas-phase and the  $\text{CO}_2$  concentration in the solid-phase decrease significantly, as shown in Figures 13a and 13b. This is because the fresh methane and hydrogen can reduce the partial pressure of  $\text{CO}_2$  in the bulk phase (the partial pressures of  $\text{CO}_2$  are 1.26 kPa at the end of Step 4 and 1.14 kPa at the end of Step 5, and the corresponding equilibrium solid concentrations are 0.13 mol/kg and 0.12 mol/kg, respectively).

Methane is lost in the first three steps. To measure the methane efficiency, we calculate the unused  $\text{CH}_4$  in one cycle as follows

$$\begin{aligned} \text{Unused CH}_4 \text{ in one cycle} &= \\ &= \frac{\text{Feed of CH}_4 \text{ in Steps 1 and 3} - \text{Consumption of CH}_4 \text{ in Step 1}}{\text{Feed of CH}_4 \text{ in Steps 1 and 3}} \\ &= 0.92. \quad (14) \end{aligned}$$

That is to say that 0.92 methane is lost compared to the methane fed. But the methane loss does not impact the viability of this system, because (1) the effluent gas from Steps 2 and 3, containing unrecovered  $H_2$  and the mixture of  $CH_4$ ,  $H_2O$ ,  $CO$ , and  $CO_2$  can be used as part of the fuel required to supply the heat of the endothermic SMR reaction, and (2)  $CH_4$  can be replaced by steam or other inert gas in Step 3 (Hufton et al., 1999). If  $CH_4$  is replaced by steam in Step 3, the unused  $CH_4$  in one cycle is

Unused  $CH_4$  in one cycle =

$$\frac{\text{Feed of } CH_4 \text{ in Step 1} - \text{Consumption of } CH_4 \text{ in Step 1}}{\text{Feed of } CH_4 \text{ in Step 1}} = 0.36. \quad (14a)$$

In conclusion, under cyclic steady-state operation conditions,  $H_2$  product gas of 80% average purity can be continuously produced from a 2-m-long adsorptive reactor. Increasing the length of the adsorptive reactor produces higher purity  $H_2$  product gas, but the regeneration of the adsorbent is very difficult due to the pressure loss limitation. When Waldron et al. (2001) carried out experiments in a 6-m-long adsorptive reactor, 88 ~ 94% purity  $H_2$  product gas was continuously produced from the adsorptive reactor under cyclic steady-state operating conditions, where the adsorbent regeneration was carried out at subatmospheric pressure and the cyclic steady state was typically reached after 30 cycles. In our simulation, the cyclic steady state is achieved after 15 cycles, since a longer total cyclic time is used.

## Conclusions

It is found that where the adsorbent and catalyst are packed in a multifunctional reactor, the methane conversion is enhanced; a high-purity hydrogen product gas with traces of  $CO_2$  and  $CO$  can be produced at a lower temperature and lower pressure compared with the conventional steam-reforming method if the operation times are controlled. For a given system (the adsorbent, temperature, and pressure are specified), increasing the packing density, the adsorbent/catalyst ratio, and the residence time in the adsorptive reactor can improve the methane conversion, and favor the production of high-purity hydrogen.

The simulated results are in reasonable agreement with experimental data from the literature (Hufton et al., 1999; Ding and Alpay, 2000b) for sorption-enhanced steam-methane reforming. The sorption-enhanced steam-methane reforming is carried out at a high pressure ( $P_H = 445.7$  kPa) and the regeneration of the adsorbent that was saturated with  $CO_2$  is carried out at a low pressure ( $P_L = 125.7$  kPa). Under the cyclic steady-state conditions, 80% average purity  $H_2$  product gas with traces of  $CO_2$  and  $CO$  can be continuously produced from a 2-m-long adsorptive reactor. Increasing the length of the adsorptive reactor produces a higher purity  $H_2$  product gas, but regeneration of the adsorbent then becomes very difficult due to the pressure loss limitation. Therefore,

the regeneration method needs to be improved. The total cyclic time is higher than that in the conventional pressure swing adsorptive reactor, and so the cyclic steady state is quickly reached.

## Acknowledgments

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## Notation

- $A$  = cross-sectional area of the reactor,  $m^2$
- $b_{CO_2}$  = Langmuir model constant for component  $CO_2$ ,  $Pa^{-1}$
- $\bar{C}$  = total molar concentration in the bulk phase,  $mol/m^3$
- $C_{fi}$  = gas-phase concentration of component  $i$  in the feed,  $mol/m^3$
- $C_i$  = molar concentration of gas-phase component  $i$ ,  $mol/m^3$
- $C_{pg}$  = gas-phase heat capacity,  $J/mol \cdot K$
- $C_{ps}$  = solid-phase heat capacity,  $J/kg \cdot K$
- $C_{vg}$  = heat capacity of gas phase at constant volume,  $J/mol \cdot K$
- $d_p$  = particle diameter,  $m$
- $D_m$  = molecular diffusivity,  $m^2/s$
- $D_p$  = combination of molecular and Knudsen diffusivity,  $m^2/s$
- $D_L$  = axial dispersion coefficient,  $m^2/s$
- $m_{CO_2}$  = Langmuir model constant for component  $CO_2$ ,  $mol/kg$
- $\bar{M}$  = average molar weight of the gas mixture,  $kg/mol$
- $n$  = cycle number
- $k_{CO_2}$  = LDF mass-transfer coefficient,  $s^{-1}$
- $k_g$  = gas-phase thermal conductivity,  $J/m \cdot s \cdot K$
- $k_i$  = rate constant of reaction  $i$ ,  $i = 1, 2$ :  $mol \cdot Pa^{0.5}/kg \cdot cat \cdot s$ ,  
 $i = 3$ :  $mol/kg \cdot cat \cdot s$
- $k_p$  = particle thermal conductivity,  $J/m \cdot s \cdot K$
- $k_z$  = effective thermal conductivity,  $J/m \cdot s \cdot K$
- $K_D$  = Ergun equation coefficient,  $N \cdot s/m^4$
- $K_V$  = Ergun equation coefficient,  $N \cdot s^2/m^5$
- $L$  = reactor length,  $m$
- $P$  = local total pressure,  $Pa$
- $P_i$  = partial pressure of gas-phase component  $i$ ,  $Pa$
- $P_H$  = high pressure,  $Pa$
- $P_L$  = low pressure,  $Pa$
- $\bar{q}_i$  = solid-phase concentration for component  $i$  (average over an adsorbent particle),  $mol/kg$
- $q^*$  = equilibrium solid-phase concentration,  $mol/kg$
- $r_i$  = formation or consumption rate of component  $i$ ,  $mol/kg \cdot cat \cdot s$
- $r_p$  = radius of the adsorbent,  $m$
- $R$  = universal gas constant,  $J/mol \cdot K$
- $R_0$  = inner radius of the reactor,  $m$
- $R_j$  = reaction rate defined by Eq. 1,  $mol/kg \cdot cat \cdot s$
- $t$  = time,  $s$
- $t_i$  = operational time for step  $i$ ,  $s$
- $T$  = temperature in bulk gas-phase,  $K$
- $T_f$  = feed gas temperature,  $K$
- $T_w$  = wall temperature,  $K$
- $u$  = superficial velocity,  $m/s$
- $u_{i0}$  = initial superficial velocity for step  $i$ ,  $m/s$
- $U$  = overall bed-wall heat-transfer coefficient,  $J/m^2 \cdot K$
- $v_{ij}$  = stoichiometric coefficient of component  $i$  in reaction  $j$
- $y_{fi}$  = gas-phase mole fraction of component  $i$  in the feed
- $y_i$  = gas-phase mole fraction of component  $i$
- $z$  = axial coordinate in bed,  $m$

## Greek letters

- $\alpha$  = mass ratio of adsorbent and catalyst in the packed bed  
 $\epsilon_b$  = bed porosity  
 $\epsilon_p$  = adsorbent porosity  
 $\epsilon_t$  = total bed porosity  
 $\eta_i$  = catalyst effectiveness factor  
 $\zeta = z/L$   
 $\mu$  = viscosity of fluid, kg/m $\cdot$ s  
 $\rho_{ad}$  = mass of adsorbent per bed volume, kg/m $^3$   
 $\rho_b$  = bulk packing density of the adsorbent and catalyst, kg/m $^3$   
 $\rho_{cat}$  = mass of catalyst per bed volume, kg/m $^3$   
 $\rho_g$  = gas-phase density, kg/m $^3$   
 $\rho_p$  = adsorbent pellet density, kg/m $^3$   
 $-\Delta H_{adi}$  = adsorption heat of component  $i$  (on the adsorbent surface), J/mol  
 $\Delta H_{Ri}$  = reaction heat of reaction  $i$ , J/mol

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